Nanoelectromechanical spin injector with large current signal at room temperature

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We propose theoretically a nanoelectromechanical spin injector based on shuttle transport mechanism, which consists of a movable island suspending between a ferromagnetic-metal (FM) source and a semiconductor (SC) drain. It is found that in the Coulomb blockade region, the spin polarization of shuttle-assisted currents is as high as that of the FM source, and the shuttling current into the SC drain is much larger than the tunneling current. It is further shown that the high-efficiency spin injector with large current signal works well at room temperature.

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During the past decade, desire for manipulation of spin states in semiconductor (SC) devices triggered rapid development of spintronics. A prerequisite for the implementation of spintronic devices is to electrically inject spin-polarized currents into the SC. From the viewpoint of application, a qualified spin injector must satisfy at least three requirements: (i) high spin-injection efficiency, (ii) large injected current signal, and (iii) operational performance at room temperature. A ferromagnetic-metal/semiconductor (FM/SC) junction with Ohmic contact has very low spin-injection efficiency due to conductivity mismatch.¹ The technology of introducing either Schottky barrier² or tunnel barrier³ was found to be effective in enhancing spin-injection efficiency, yielding above 30% spin polarization of currents for Fe spininjection source. The spin-injection efficiency can be enhanced further by increasing the height of the contact barrier.⁴ However, the current transmitted through a tunnel junction with high barrier is usually very weak, and the higher the barrier, the weaker one finds the current signal into the SC. This to a certain extent impedes the experimental detection by common electroluminescence methods and limits its applications in spin electronic components. Even for spin-polarized hot electron transports, $5,6$ $5,6$ the transmitted current is also smaller than the emitting current by a factor of orders of magnitude. Moreover, most of the experiments keep high injection efficiency only at low temperatures, and the spin-injection efficiency decreases dramatically at higher temperatures[.7](#page-3-7) Jiang *et al.*[8](#page-3-8) pointed out that the light emission efficiency drops rapidly at high temperatures, so that a larger current is required to maintain a strong enough electroluminescence signal at high temperatures. Therefore, it is highly desirable to search for high-efficiency spin injectors with large current signals, especially at room temperature.⁹

Quantum dots (QDs), which exhibit some interesting quantum effects, such as Coulomb blockade (CB), have become powerful tools to control the quantum states and transport properties of electrons. The FM/SC tunnel junction with QD as a central spacer was suggested to achieve a large spin polarization of current via spin-dependent resonant tunneling.¹⁰ However, the difficulty of small transmitted currents still exists, as the tunneling current is always small even in the resonant case. A possible way of increasing the current through a tunnel junction with QD is to replace the static QD with a movable QD oscillating between two conducting leads. In such a nanoelectromechanical (NEM) system, it is possible for electrons to be directly shuttled from one lead to the other, called the shuttling transport.¹¹ This novel shuttling transport, which is completely different from the tunneling transport, was proposed and studied in the NEM systems with normal-metal leads $11-14$ $11-14$ and recently ap-plied to those with FM leads.^{15–[17](#page-3-14)} Shuttling transports can be realized experimentally by self-excited mechanical oscillations^{18[,19](#page-3-16)} or by driving oscillations with an additional ac voltage. $20,21$ $20,21$ The shuttling current is proportional to the oscillation frequency of the QD, independent of the tunneling rate between two leads. As a result, as soon as the shuttling mechanism dominates the electron transport of the NEM single-electron transistor, the current is greatly en-hanced, as reported in both theoretical studies^{11[–17](#page-3-14)} and experimental observations[.18](#page-3-15)[–21](#page-3-18) This result reminds us that the difficulty of small current in the FM/QD/SC junctions can be solved by the shuttling transport. However, it has not been clear if there is a high spin polarization of the shuttling current in the NEM FM/QD/SC system and if it can work well at room temperature.

In this Brief Report we clear up the questions above by taking into account an NEM FM/QD/SC system consisting of a movable QD that oscillates between an FM source and an SC drain. The shuttling transport through the QD is analyzed in the quantum regime with the help of the Wigner distribution function by combination of single-electron tunneling with mechanical degrees of freedom of the QD. Our calculations show that the spin polarization of the shuttleassisted current is high and equal to the spin polarization of the FM in the CB region. The shuttling current into the SC is greater than the tunneling current by at least 1 order of magnitude. We also study the temperature dependence of the spin polarization of the injected current and find that it is possible to realize high-efficiency spin injection with large injection current even at room temperatures. These results are robust under the change in system parameters.

Consider an NEM FM/QD/FM system with *M* as the mass of the movable QD and ω_0 as its vibration frequency. Throughout this Brief Report all the lengths are taken in unit of the zero-point oscillation amplitude $x_0 = \sqrt{\hbar / M \omega_0}$ and energies in unit of $\hbar \omega_0$. The dimensionless Hamiltonian of the system takes the form¹² $\hat{H} = \hat{H}_{FM} + \hat{H}_{SC} + \hat{H}_{OD} + \hat{H}_T$. Here $\hat{H}_{\text{FM}} = \sum_{k\sigma} \varepsilon_{k\sigma F} c_{k\sigma F}^{\dagger} c_{k\sigma F}$ is the electronic Hamiltonian of the FM source with $\varepsilon_{k\sigma F} = \varepsilon_{kF} \pm h$ as the energy of a spin- σ electron under the internal molecular field *h* and \hat{H}_{SC} $= \sum_{k\sigma} \varepsilon_{kS} c_{k\sigma S}^{\dagger} c_{k\sigma S}$ is that of the nonmagnetic SC drain with ε_{kS} as the single-electron energy of the two-dimensional (2D) electron gas. The Hamiltonian of the movable QD is given by

$$
\hat{H}_{\rm QD} = \sum_{\sigma} (\varepsilon_0 - \eta \hat{x}) d_{\sigma}^{\dagger} d_{\sigma} + \hat{H}_{\rm osc} + \hat{H}_{\rm damp}, \tag{1}
$$

where ε_0 is the single-electron energy level in the dot shifted by $\eta \hat{x}$ with $\eta = e \epsilon x_0 / \hbar \omega_0$ due to the electric field ϵ proportional to bias voltage *V* between the leads. The center-ofmass motion of the QD is assumed to be confined in a harmonic potential $\hat{H}_{osc} = (\hat{p}^2 + \hat{x}^2)/2$ with operators \hat{p} and \hat{x} standing for the dimensionless momentum and displacement of the QD oscillator, respectively. Due to thermal fluctuations, the motion of the oscillator is damped by the bosonic environment through $\hat{H}_{\text{damp}} = \sum_p \varepsilon_p b_p^{\dagger} b_p + g_p \hat{x}(b_p + b_p^{\dagger})$ with g_p as the coupling strength between the oscillator and thermal bath.²² The tunneling Hamiltonian is given by H_T = $\sum_{k\sigma} T_{F\sigma}(\hat{x}) c_{k\sigma F}^{\dagger} d_{\sigma} + \sum_{k\sigma} T_{S}(\hat{x}) c_{k\sigma S}^{\dagger} d_{\sigma} + \text{H.c.,}$ where $T_{F\sigma}(\hat{x}) = T_{F\sigma} e^{-\hat{x}/\lambda} [T_{S}(\hat{x}) = T_{S} e^{\hat{x}/\lambda}]$ is the tunnel matrix element between the FM (SC) and QD with λ as the tunneling length. The exponential \hat{x} dependence in $T_{F\sigma}(\hat{x})$ and $T_{S}(\hat{x})$ indicates that the transport properties are considerably modulated by the electromechanical motion of the QD.

The tunneling rate between the FM source and QD is $\Gamma_{F\sigma}(\hat{x}) = \Gamma_{F\sigma}e^{-2\hat{x}/\lambda}$ with $\Gamma_{F\sigma} = 2\pi\Omega_{F\sigma}|T_{F\sigma}(0)|^2$, where $\Omega_{F\uparrow}$ (Ω_F) is the density of states of the majority (minority) spin subband in the FM source. The spin polarization of the FM is defined as $P = (\Omega_{F\uparrow} - \Omega_F) / (\Omega_{F\uparrow} + \Omega_F)$, so that $\Gamma_{F\uparrow(\downarrow)}(\hat{x}) = \Gamma_{F0}(1 \pm P) e^{-2\hat{x}/\lambda}/2$, where Γ_{F0} describes the tunneling rate in the absence of internal magnetization. The tunneling rate between the QD and SC drain is $\Gamma_s(\hat{x}) = \Gamma_{s0}e^{2\hat{x}/\lambda}$ with $\Gamma_{S0} = 2\pi \Omega_S |T_S(0)|^2$, where Ω_S is the density of states in the nonmagnetic SC. Define an asymmetry factor as χ $=\Gamma_{\text{SO}}/\Gamma_{\text{F0}}$. Since Ω_{S} for a 2D electron gas is usually much smaller than $\Omega_{F\uparrow}$ and $\Omega_{F\downarrow}$, we have $\Gamma_{S0} \ll \Gamma_{F0}$ or $\chi \ll 1$.

The full Hamiltonian is treated by adopting the standard quantum master equation approach.^{12[–17](#page-3-14)} In order to analyze the quantum oscillation for convenience, we employ the Wigner distribution function $W(x,p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} dy \langle x - \frac{y}{2} | \rho | x + \frac{y}{2} \rangle e^{ipy/\hbar}$ with ρ being the reduced density matrix operator by tracing out the degrees of freedom of the thermal bath and electron reservoirs in leads. When the density matrix operator ρ is mapped onto the corresponding Wigner distribution function $W(x, p)$, arguments *x* and *p* of $W(x, p)$ are numbers instead of operators. The quantum commutation rule $[\hat{x}, \hat{p}]$ is reflected in a 2*n*-order differential series^{13–[17](#page-3-14)} with respect to *p*, i.e., $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$ $\frac{(-1)^n}{(2n)!} (\frac{1}{\lambda})^{2n} \partial_{p}^{2n} W$. If we take only the *n*=0 term, the commutation relation $\lceil \hat{x}, \hat{p} \rceil$ vanishes, which corresponds to a classic picture. However, it is necessary to consider the oscillation in the quantum regime because the ultrasmall QD has a high oscillation frequency.¹²

We retain the expansion series up to the second order, i.e., the term including $\frac{1}{2}\partial_p^2$. This is a reasonable approximation in realistic cases,¹³ where λ is much greater than the amplitude of the zero-point oscillation of the QD. In subsequent discussion of the steady-state solutions, we switch from the $x-p$ coordinate system to the $A - \theta$ polar coordinate system with $x/\lambda = A \sin \theta$ and $p/\lambda = A \cos \theta$.

Following the derivation of literatures^{13[–17](#page-3-14)} and taking into account finite temperatures, $2³$ we obtain the expression for stationary currents injecting into the SC as

$$
I_{\uparrow(\downarrow)} = e \int_0^\infty W_+(A)A dA \int_0^{2\pi} d\theta \Gamma_s [(1 + G_-) f_S
$$

+ $(1 - G_- \pm 2G_{\uparrow\downarrow})(1 - f_S)/2]$ (2)

in the CB region. The Wigner distribution functions is given by $W_+(A) \approx Z^{-1} \exp[-\int_0^A dA' f(A') / D(A')]$, with $f(A) = \frac{\pi}{2\lambda} P_\theta \cos \theta G_- + A\gamma/2$, $D(A) = \frac{\gamma}{2\lambda^2} (\frac{1}{2} + \overline{N}) + \frac{1}{\lambda^4} \beta_1(A) + (\frac{\pi}{2\lambda})^2 \beta_2(A)$, and *Z* as the renormalization factor. Here *P*_{θ} is a projector defined as $P_{\theta}f(\theta) \equiv \frac{1}{2\pi} \int_0^2 \int_0^{\pi} f(\theta) d\theta$, γ is the dissipative rate²² with $\gamma \ll 1$ in the weak dissipation case, $\overline{N} = 1/[\exp(\frac{\hbar \omega_0}{k_B T}) - 1]$ is the average phonon number for the damping reservoir at finite temperature *T*, and $\beta_1(A)$ and $\beta_2(A)$ are those in Ref. [17](#page-3-14) extending to finite temperatures. $G_-(A, \theta)$ and $G_{\uparrow\downarrow}(A, \theta)$ in Eq. ([2](#page-1-0)) are calculated with the help of the following self-consistent equations:

$$
\partial_{\theta} G_{-} = -\left[\Gamma_{F+}(1+f_F) + \Gamma_S(1+f_S)\right]G_{-} + 2\Gamma_{F-}(1-f_F)G_{\uparrow\downarrow} + \left[\Gamma_{F+}(1-3f_F) + \Gamma_S(1-3f_S)\right],
$$
(3)

$$
\partial_{\theta}G_{\uparrow\downarrow} = \frac{1}{2}\Gamma_{F-}(1+f_F)G_{-} - [\Gamma_{F+}(1-f_F) + \Gamma_{S}(1-f_S)]G_{\uparrow\downarrow} + \frac{1}{2}\Gamma_{F-}(3f_F - 1),
$$
\n(4)

together with the periodic boundary conditions $G_{\uparrow\downarrow(-)}(A,\theta+2\pi)=G_{\uparrow\downarrow(-)}(A,\theta)$. Here we denote $\Gamma_{F\pm}$ $=\left(\Gamma_{F\uparrow} \pm \Gamma_{F\downarrow}\right) e^{-(2A\sin(\theta+\eta/\lambda))}/2$ and $\Gamma_{S}=\Gamma_{S0}e^{(2A\sin(\theta+\eta/\lambda))}/2$ for briefness and the Fermi distribution function $f_{F(S)}$ $=1/[1+\exp(\frac{\varepsilon_0-\mu_{F(S)}}{k_BT})]$ with electrochemical potential $\mu_{F(S)}$ $= \pm eV/2(e>0).$

We are interested in how the electronic current injecting from the FM to SC responds to the dynamical coupling of the electronic and vibrational degrees of freedom. Some parameters for calculations are taken from the experiment:¹⁸ $\hbar \omega_0$ =5 meV and x_0 =10⁻³ nm. Spin-dependent stationary current components I_1 and I_1 vs electric field in the CB region are shown in Fig. [1.](#page-2-0) For either spin channel, there appears an abrupt transition in current at certain threshold of the applied electric field on either side of which $I_1(I_1)$ exhibits a saturated plateau. It is found that the current on the higher plateau is at least an order greater than that on the lower plateau. Such a difference arises from different transport mechanisms. For electric field smaller than the threshold, the amplitude probability distribution of the oscillator,

FIG. 1. (Color online) Injected current vs electric field in the CB region at *T*=0. Inset: spin-dependent average electron occupation $n_{\uparrow(\downarrow)}$ on the QD as a function of ϵ . n_{\uparrow} (solid line) and n_{\downarrow} (dashed line) correspond to the label in the left and the ratio $n_{\uparrow}/n_{\downarrow}$ (dotted line) to the label in the right. The parameters used are Γ_{F0} =0.18, $\chi=0.1, \ \gamma=0.003, \ P=0.4, \text{ and } \lambda=5.$

 $AW_+(A)$, shows a peak around the origin with $A=0$, and so the transport of electrons is tunnel type. For large electric field, the maximum of $AW_+(A)$ is in the classic limit cycle A_{c} [determined by conditions $f(A_{cl}) = 0$ and $f'(A_{cl}) > 0$],^{[13](#page-3-20)[,14](#page-3-12)[,17](#page-3-14)} suggesting shuttle transport with a stable amplitude. The shuttling current is determined only by the oscillation frequency $f = \omega_0 / 2\pi$ of the QD and usually much greater than the tunneling current. The spin injection based on the shuttling transport suggests a way out of the difficulty of small current signal. In the vicinity of the threshold field, both tunneling and shuttling transport mechanisms contribute to the currents, with the oscillator jumping between two stable amplitudes and presenting a bistable state.

The spin-injection efficiency ξ can be defined as $\xi = (I_{\uparrow} - I_{\downarrow})/(I_{\uparrow} + I_{\downarrow})$. From Fig. [1,](#page-2-0) it is found that in either tunneling or shuttling regime, the current is strongly spin dependent, and ξ is always equal to the spin polarization $P=40\%$ of the FM source. The result of $\xi = P$ has been also obtained by Souza *et al.*[24](#page-3-22) in a similar tunnel junction with a stationary QD and in the CB region. The present calculations show that the relation of $\xi = P$ is suitable not only for the tunneling regime but also for the shuttling regime. Furthermore, compared with the result of $I'_{\sigma} = \frac{e\Gamma_{\text{S0}}}{\Gamma_{\text{S0}} + 2\Gamma_{F0}} \Gamma_{F\sigma}$ derived from Eq. (18) of Ref. [24,](#page-3-22) we obtain $I_{\sigma} = \beta I_{\sigma}^{T}$ in the tunneling regime, where the prefactor of $\beta \ge 1$ arises from the vibration-assisted tunneling.

The interesting finding of $\xi = P$ in the shuttling regime can be understood by the following argument. Define $n_{\uparrow(\downarrow)} = \int_0^\infty W_+(A)A dA \int_0^{2\pi} d\theta [1 - G_-(A, \theta) \pm 2G_{\uparrow\downarrow}(A, \theta)]/4$ as the average occupation¹³ of the spin-up (spin-down) electron on the QD in an oscillation period. n_{\uparrow} (solid line) and n_{\downarrow} (dashed line) are plotted as functions of electric field in the inset of Fig. [1.](#page-2-0) It is found that, in the shuttling regime for the electric field above 0.82 V/nm, the ratio n_1/n_1 (dotted line) approaches a constant value of 2.33, which is just equal to

FIG. 2. (Color online) Injected current vs electric field for three different temperatures. The other parameters are the same as in Fig. [1.](#page-2-0)

 $(1+P)/(1-P)$. The calculated shuttling currents I_{σ} can be fitted very well with formula $I_{\sigma} = 2en_{\sigma}f$, resulting in a large spin-injection efficiency of $\xi = P$. In the shuttling regime, although I_{σ} is proportional to the occupation n_{σ} , the total current $I_1 + I_2 = ef$ depends only on the oscillation frequency due to $n_1 + n_1 = 1/2$. This feature stems from the fact that in each round trip of the shuttling, only one electron travels by the QD from the FM source to the SC drain, and the return QD is free of electron. The ratio $n_{\uparrow}/n_{\downarrow}$ in loading probability between the spin-up and spin-down electrons is determined by the spin polarization of the FM source. It is noted that the variation in parameters $\Gamma_{F(S)0}$ is mainly to change the electric filed threshold for the tunneling-shuttling transition but not to change the I_{σ} (or n_{σ}) in the shuttling regime, which is quite different from that in the tunneling regime.

In Fig. [2](#page-2-1) the spin-resolved currents are plotted as functions of electric field for three different temperatures. An increase in temperature leads to a slower crossover from the tunneling to shuttling regime, yielding a broadened transition region. While the tunneling currents at low electric fields somewhat increase with temperature, the shuttling current plateaus at larger electric fields remain unchanged. Since both I_1 and I_1 in the shuttling regime do not have any change, the NEM spin injector with high injection efficiency $\xi = P$ still performs very well at room temperature.

We also examine the shuttling transport in the free region, where the QD energy level may be occupied by two electrons and the two spin-channel model holds. The spin polarization of the shuttling current is found to be completely suppressed. This is because in the two spin-channel model, a pair of electrons with opposite spins are shuttled together from the FM to SC in each cycle. Therefore, the system must operate in the CB region to achieve high-efficiency spin injection. Fortunately, with today's available experimental technology, it is possible to ensure the CB condition even at room temperature. For example, as reported by Park *et al.*, [18](#page-3-15)

the charging energy of a C_{60} single-molecule transistor is greater than 270 meV, much higher than the energy scale of room temperature.

In summary the nanoelectromechanical spin injector is proposed to realize high-efficiency spin injection from an FM to an SC based on the QD shuttle transport in the CB region. It is found that the spin polarization of the injected current in the SC drain is equal to the spin polarization in the FM source. At the same time, the shuttling current into the SC is much greater than the tunneling current. As a result, the present spin injector based on the QD shuttle transport mediates a settlement between the high spin polarization and

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large signal of the injected current. In particular, such a spin injector can work very well at room temperature, which has inviting prospects for spintronic applications.

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